

## Worksheet for September 19

Problems marked with an asterisk are to be placed in your math diary.

(1.\*) Show that the largest rectangular box having a fixed surface area must be a cube. Why is your answer a maximum, and not a minimum?

(2\*.) Consider the function  $f(x, y) = \frac{1}{x^2+y^2+1}$ .

- (i) Find the best quadratic approximation to  $f(x, y)$  at  $(0,0)$ .
- (ii) Use the limit definition to show that your answer in (i) is actually the best quadratic approximation to  $f(x, y)$  at  $(0,0)$ .
- (iii) Based upon the formulas for best linear approximation and best quadratic approximation for a function  $f(x, y)$ , write a formula for what you think gives the best cubic, or third order, approximation for a general  $f(x, y)$  at  $(a, b)$  and apply this to  $f(x, y) = \frac{1}{x^2+y^2+1}$  at the point  $(0,0)$ .

(3.) Find and classify the critical points for  $f(x, y, z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$ .